

## SET Homework 5

Solve Problems 7.21 and Problems 12.9 and 2 from the other HW problems. Due time: Mar 17, Tuesday 8:15.

You can get extra credit for the solutions of extra problems. You can submit the solutions of at most 2 extra problems weekly. Groups can submit joint solutions for shared credit. The extra problems do not have due time.

### Compulsory HW problems

**Problem 7.21.** Prove that every poset  $\langle Q, \leq \rangle$  contains a maximal antichain using the Zorn lemma.

**Problem 12.9.** Let  $\mathcal{Q} = \langle Q, \leq \rangle$  be a poset. Using Transfinite Recursion prove that  $\mathcal{Q}$  contains a maximal antichain.

### HW problems

**Problem 5.6.** Assume that  $A \subseteq \mathbb{R}$  is well-ordered (in the usual ordering of the reals). Prove that  $A$  is countable.

**Problem 5.7.** Let

$$\mathbb{A} = \{A \subseteq \mathbb{Q} : A \text{ is well-ordered (in the usual order of the reals)}\}.$$

Prove that  $|\mathbb{A}| = |\mathbb{R}|$ .

**Problem 5.16.** If the well-ordered set  $\langle A, \leq \rangle$  is similar to  $\langle B, \leq \rangle$ , then the order preserving bijection between them is unique.

### Extra HW problems

**Problem 5.3.** Consider the set of all polynomials in one variable  $x$  whose coefficients are natural numbers. Order it as follows: a polynomial  $P$  is greater than a polynomial  $Q$  if  $P(x) > Q(x)$  for all sufficiently large  $x$ . Prove that this condition defines a well-ordering.

**Problem 5.4.** Dr. Faust signed a contract with Mephistopheles: every day he gives Mephistopheles a coin, and in exchange gets any set of coins he wishes, but all the coins must be of a lesser value (coins come in a finite number of denominations). Dr. Faust is not allowed to change (or earn) money anywhere else, but can spend money as he wishes. Dr. Faust loses when he has no more coins left. Prove that Mephistopheles will eventually win, whatever set of coins Dr. Faust initially had.