SET Homework 4

Solve 6 from the 9 HW problems. Due time: Mar 10, Tuesday 8:15.

You can submit more than 6 solutions. The best 6 count.

You can get extra credit for the solutions of extra problems. You can submit the solutions of at most 2 extra problems weekly. Groups can submit joint solutions for shared credit. The extra problems do not have due time.

HW problems

Problem 10.6. If $A_0, A_1, \ldots$ are infinite sets, then there are pairwise different objects $x_0, x_1, \ldots$ such that $x_{2i} \in A_i$ and $x_{2i+1} \in A_i$ for all $i \in \mathbb{N}$.

Problem 10.14. Prove that if $A_0, A_1, \ldots$ are infinite sets, then there is a set X such that $A_n \cap X \neq \emptyset$ and $A_n \setminus X \neq \emptyset$ for each $n \in \mathbb{N}$.

Problem 10.8. There are infinitely many pairwise disjoint infinite subsets $B_0, B_1, \ldots$ of ω.

Problem 10.11. If $C_0, C_1, \ldots$ are infinite sets, then there are pairwise disjoint sets $D_0, D_1, \ldots$ such that $D_n \subseteq C_n$ for $n \in \mathbb{N}$, moreover $\bigcup_{n\in\mathbb{N}} D_n = \bigcup_{n\in\mathbb{N}} C_n$.

Problem 6.34. There is a function $f : \mathbb{R} \to \mathbb{R}$ such that for all $g \in C(\mathbb{R})$ there is $x \in \mathbb{R}$ such that $f(x) = g(x)$.

Problem 5.2. (1) Find the proper initial segments A of $\mathbb{R}$ which can not be obtained as $A = \mathbb{R}_{<x}$.
(2) Find the proper initial segments B of $\mathbb{Z}$ which can not be obtained in as $B = \mathbb{Z}_{<m}$.

Problem 5.5. Consider the set of all finite strings of English letters with lexicographical order. Is this ordered set well-ordered?

Problem 5.9. Consider the ordered set $\langle [0,1] \times [0,1], \leqslant_{lex} \rangle$. Prove that if $A \subseteq [0,1] \times [0,1]$ is countable, then there are $x, y \in [0,1] \times [0,1]$ such that $x <_{lex} y$, but there is no $a \in A$ with $x <_{lex} a <_{lex} y$.

Problem 5.14. Assume that $\langle A, \leqslant \rangle$ is a well ordered set. If $g : A \hookrightarrow\!\!\!\!\rightarrow A$ is an order preserving bijection, then $g(x) = x$ for all $x \in A$.

Extra HW problems

Problem 6.31. A point $x \in \mathbb{R}$ is called a **maximum point** for a function $f : \mathbb{R} \to \mathbb{R}$ if there exists some $\varepsilon > 0$ such that $f(x) > f(x+h)$ for any h such that $|h| < \varepsilon$ and $h \neq 0$. Prove that the set of all maximum points (for any function f) is countable.

Problem 6.35. There is a function $f : \mathbb{R} \to \mathbb{R}$ such that for all $g \in C(\mathbb{R})$ we have $|\{x \in \mathbb{R} : f(x) = g(x)\}| = |\mathbb{R}|$.

Problem 10.12. If $A_0, A_1, \ldots$ are infinite sets, then there are pairwise disjoint infinite sets $E_0, E_1, \ldots$ such that $E_n \subseteq A_n$ for $n \in \mathbb{N}$, and $\bigcup_{n\in\mathbb{N}} E_n = \bigcup_{n\in\mathbb{N}} A_n$.