SET Homework 2

Solve 6 from the 9 HW problems. Due time: Feb 25, Tuesday 8:15.

You can submit more than 6 solutions. The best 6 count.

You can get extra credit for the solutions of extra problems. You can submit the solutions of at most 2 extra problems weekly. Groups can submit joint solutions for shared credit. The extra problems do not have due time.

HW problems

Problem 2.4. Do there exist sets A, B and C such that $A \cap B \neq \emptyset$, $A \cap C = \emptyset$ and $(A \cap B) - C = \emptyset$?

Problem 2.6. Prove or disprove:

(a) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$ for each sets A and B.
(b) $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$ for each sets A and B
(c) $\mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B)$ for each sets A and B.

Problem 2.11. Prove the following distributive law:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Problem 2.9. Prove Theorem 2.2(4): for $\mathcal{A} \neq \emptyset$

$$C - \bigcap \mathcal{A} = \bigcup \{C - X : X \in \mathcal{A}\}.$$

Problem 3.11. Assume that $\mathcal{F}$ is a set. Prove that $\bigcup \mathcal{F}$ is an injective function iff $f \cup g$ is an injective function for each $f \in \mathcal{F}$ and $g \in \mathcal{F}$. (In your solution you can use the statement of Problem 3.10)

Problem 1.39. Let $L(x, y)$ be the statement "x *loves* y," where the "domain" for both x and y consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody loves Jerry.

b) Everybody loves somebody.

c) There is somebody whom everybody loves.

d) Nobody loves everybody.

e) There is somebody whom Lydia does not love.

f) There is somebody whom no one loves.

g) there is exactly one person whom everybody loves. □

Problem 4.5. Assume that R is a linear order on a set A and S is an equivalence relation on A.

Is it true that $R \cap S$ is not a linear order on A?

Problem 4.6. Assume that R is a partial order on a set A, and S is an equivalence relation on A.

Is it true that $R \cap S$ is a partial order on A?

Problem 4.1. Prove that for each poset $\langle Q, \preceq \rangle$ there is a set $\mathcal{A} \subseteq \mathcal{P}(Q)$ such that $\langle Q, \preceq \rangle$ and $\langle \mathcal{A}, \subseteq \rangle$ are order isomorphic (i.e. there is a bijection $F : Q \to \mathcal{A}$ such that $q_0 \preceq q_1$ iff $f(q_0) \subseteq f(q_1)$ for each $q_0, q_1 \in Q$)

Extra HW problems

Problem 12.2. Write $\langle x, y \rangle_2$ for $\{\{x\}, \{\{y\}\}\}$.

Is it true that $\langle x, y \rangle_2 = \langle a, b \rangle_2$ implies $x = a$ and $y = b$?

Problem 1.44. Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Explain your reasoning.

Problem 1.39. Let $L(x, y)$ be the statement "x *loves* y," where the "domain" for both x and y consists of all people in the world. Use quantifiers to express each of these statements.

h) There are exactly two people whom Lynn loves.

i) Everyone loves himself or herself.

j) There is someone who loves no one besides himself or herself. □