On properties of families of sets

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If κ is an infinite cardinal, $\mathcal{A} \subset [\kappa]^{\kappa}$, $|\mathcal{A}| \leq \kappa$, then $\mathcal{A}$ has property B.

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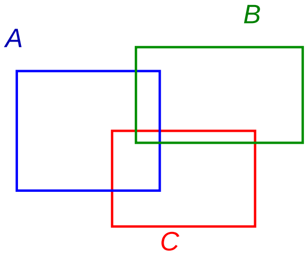

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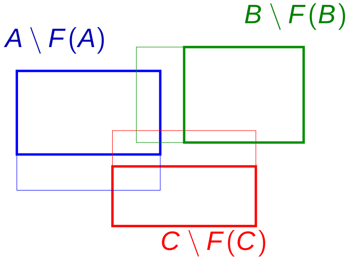

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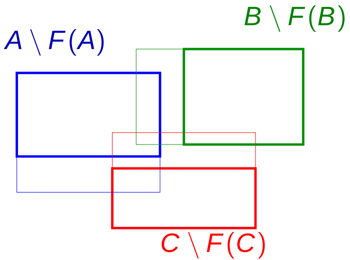

If $\mathcal{A} \subset [X]^{\mu}$ is μ**-ED** then there is a map $f : X \to \mu$ such that

$f \restriction A$ is almost 1-to-1 for all $A \in \mathcal{A}$,

and so $\mathcal{A}$ has **property B** .

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Theorem (Komjáth)

If every ω-almost disjoint subfamily of $[\lambda]^{\omega_1}$ has the transversal property
then every ω-almost disjoint subfamily of $[\lambda]^{\omega_1}$ is ω-essentially disjoint*.*

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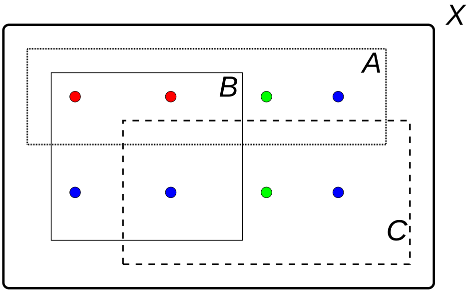

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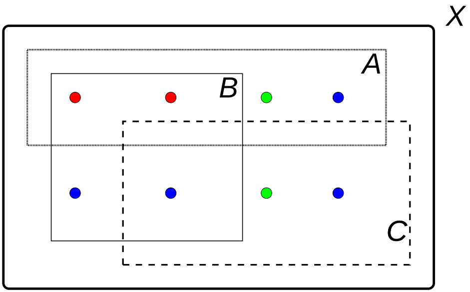

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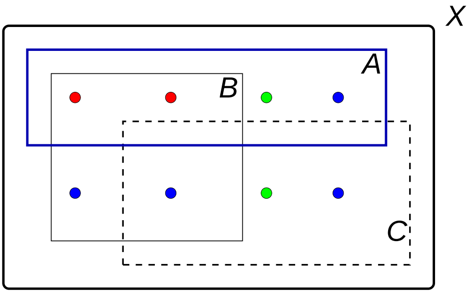

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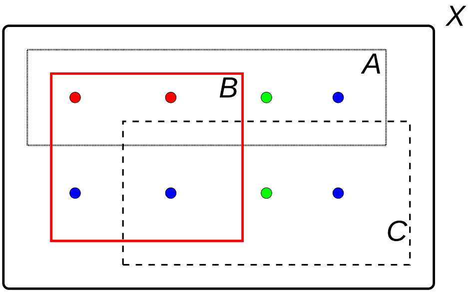

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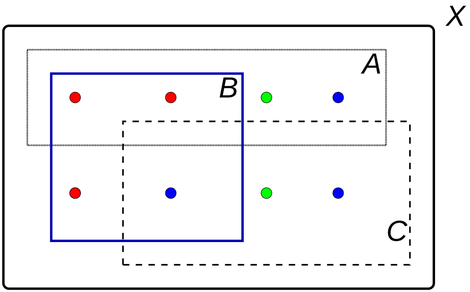

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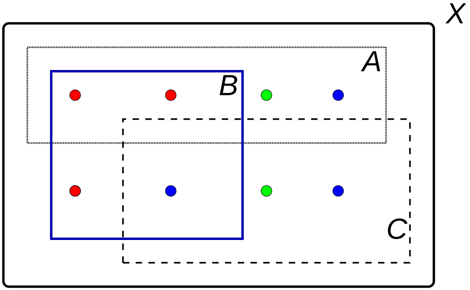

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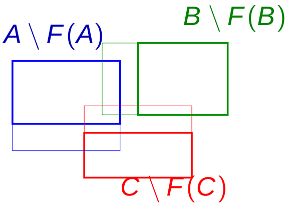

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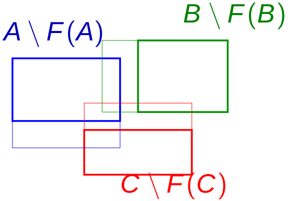

- There is $f : \lambda \to \mu$ such that $f \restriction A$ is almost 1-to-1 for all $A \in \mathcal{A}$. .

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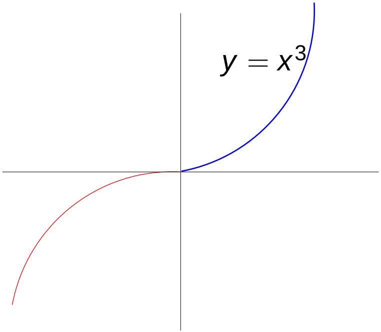

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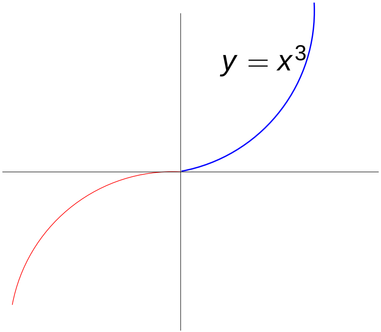

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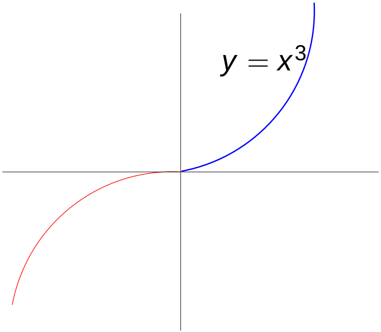

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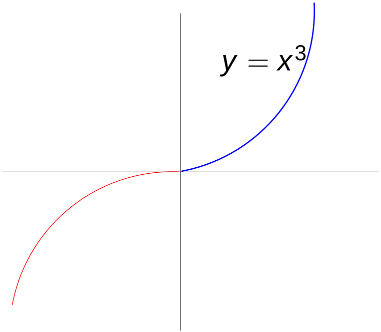

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$$\chi_{\mathrm{CF}}([\omega_m]^\omega, d\text{-a.d.}) \leq \left\lfloor \frac{(m+1)(d-1)+1}{2} \right\rfloor + 2.$$

If GCH holds, and if $d = 2$ or d is odd then

$$\left\lfloor \tfrac{(m+1)(d-1)+1}{2} \right\rfloor + 1 \leq \chi_{\mathrm{CF}}([\omega_m]^\omega, d\text{-a.d.})$$

- **Do we really need ω colors?**

Theorem

Yes, we need: $\chi_{\mathrm{CF}}([\beth_\omega]^\omega, 2\text{-}\textbf{a.d.}) = \omega$.

The Zoo of the properties of families of sets

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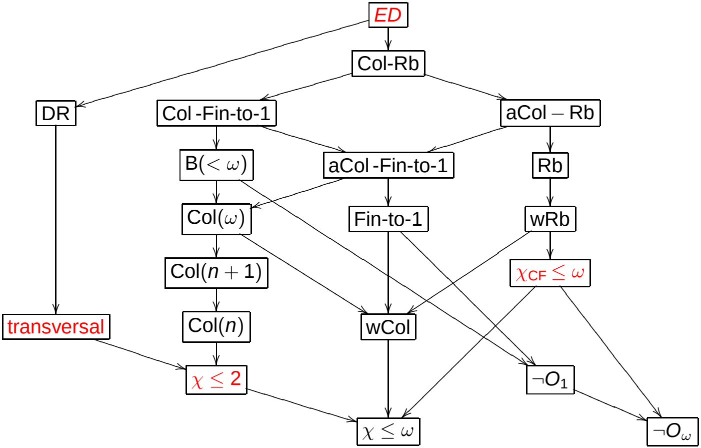
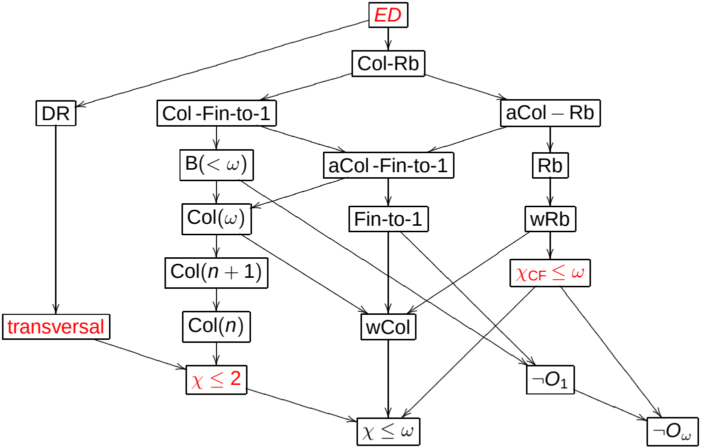
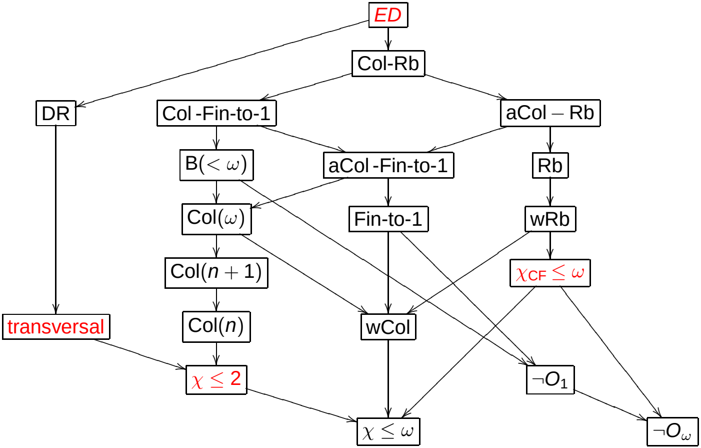


The Zoo of the properties of families of sets



Program: Separate the properties in the previous diagram!

The Zoo of the properties of families of sets



Program: Separate the properties in the previous diagram!

The assumption that

every μ-a.d. family $\mathcal{A} \subset [\lambda]^\kappa$ has property Φ

does not imply that

every μ-a.d. family $\mathcal{A} \subset [\lambda]^\kappa$ has property Ψ.

Thank you!

http://www.renyi.hu/∼soukup